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If $a, b, c > 0$ and $abc = 1$, then :

$$\sum_{cyc} \frac{a^2 + bc}{a + b} \geq 3$$

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$$\sum_{cyc} \frac{a^2 + bc}{a + b} = \sum_{cyc} \frac{a^2 - b^2 + b^2}{a + b} + \sum_{cyc} \frac{bc}{a + b} =$$

$$= \sum_{cyc} \frac{(a + b)(a - b)}{a + b} + \sum_{cyc} \frac{b^2 + bc}{a + b} =$$

$$= \sum_{cyc} (a - b) + \frac{b(b + c)}{a + b} + \frac{c(c + a)}{b + c} + \frac{a(a + b)}{c + a} \overset{A-G}{\geq}$$

$$\geq 3\sqrt[3]{\frac{abc(b + c)(c + a)(a + b)}{(a + b)(b + c)(c + a)}} \overset{abc=1}{=} 3 \therefore \sum_{cyc} \frac{a^2 + bc}{a + b} \geq 3$$

$$\forall a, b, c > 0 \,\big|\, abc = 1, \text{"} = \text{"} \text{ iff } a = b = c = 1 \text{ (QED)}$$