ROMANIAN MATHEMATICAL MAGAZINE

$$\text{If } a, b, c > 0 \text{ and } \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=3, \text{ then :}$$

$$\sum_{cyc}\left(\sqrt{3b-1}\cdot\sqrt{3c-1}\right)\geq 6$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc}\left(\sqrt{3b-1}.\sqrt{3c-1}\right)\overset{3=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{=}$$

$$\sum_{cyc}\left(\sqrt{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)b-1}.\sqrt{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)c-1}\right)=\sum_{cyc}\sqrt{\frac{b(c+a)}{ca}.\frac{c(a+b)}{ab}}$$

$$\overset{A-G}{\geq} 3.\sqrt[3]{\frac{(a+b)(b+c)(c+a)}{abc}}\overset{Cesaro}{\geq} 3.\sqrt[3]{8}$$

$$\therefore \sum_{cyc}\left(\sqrt{3b-1}.\sqrt{3c-1}\right)\geq 6\ \forall\, a,b,c>0,''='' \text{ iff } a=b=c=1\ (\text{QED})$$