

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0, n \in \mathbb{N}, n > 2$ then:

$$\sum \frac{b+c}{a + \sqrt[n]{2^{n-1}(b^n + c^n)}} \leq 2$$

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Solution by Tapas Das-India

$$\frac{b^n + c^n}{2} \stackrel{CBS}{\geq} \frac{(b+c)^n}{2^n}$$

$$2^{n-1}(b^n + c^n) \geq (b+c)^n$$

$$\sqrt[n]{2^{n-1}(b^n + c^n)} \geq b+c \quad (1)$$

$$\sum \frac{b+c}{a + \sqrt[n]{2^{n-1}(b^n + c^n)}} \stackrel{(1)}{\leq} \sum \frac{b+c}{a+b+c} = \frac{2(a+b+c)}{a+b+c} = 2$$

Equality holds for $a = b = c$