ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $abc=1$ then:

$$\sum\sqrt{1+8a^2}\ge\frac{7}{3}(a+b+c)+2$$

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*Solution by Tapas Das-India*

$$1+8a^2=1+a^2+a^2+a^2+a^2+a^2+a^2+a^2+a^2\overset{CBS}{\ge}\frac{(1+8a)^2}{9}\quad(1)$$

$$\sum\sqrt{1+8a^2}\overset{(1)}{\ge}\sum\frac{1+8a}{3}=\frac{8}{3}(a+b+c)+1=$$

$$=\frac{7}{3}(a+b+c)+\frac{1}{3}(a+b+c)+1\ge$$

$$\overset{AM-GM}{\ge}\frac{7}{3}(a+b+c)+\sqrt[3]{abc}+1=\frac{7}{3}(a+b+c)+2\ (as\ \ abc=1)$$

$$Equality\ for\ a=b=c=1$$