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If $a, b, c > 0,\ abc = 1, n \in N$ then:

$$\sum \frac{a^{n+2} + b^{n+2}}{a^{n+1}b^{n+1}(a^n + b^n)} \geq 3$$

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Solution by Tapas Das-India

$$\frac{a^{n+2} + b^{n+2}}{a^{n+1}b^{n+1}(a^n + b^n)} \overset{CBS}{\geq} \frac{\frac{(a^n+b^n)(a^2+b^2)}{2}}{a^{n+1}b^{n+1}(a^n + b^n)} = \frac{a^2 + b^2}{2} =$$

$$= \frac{\frac{a^2+b^2}{2}}{a^{n+1}b^{n+1}} \overset{AM-GM}{\geq} \frac{ab}{a^{n+1}b^{n+1}} = \frac{1}{a^n b^n} (1)$$

$$\sum \frac{a^{n+2} + b^{n+2}}{a^{n+1}b^{n+1}(a^n + b^n)} \overset{(1)}{\geq} \sum \frac{1}{a^n b^n} \overset{AM-GM}{\geq} 3\left(\frac{1}{abc}\right)^{\frac{2n}{3}} = 3\ (abc = 1)$$

$$Equality\ holds\ for\ a = b = c = 1$$