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If $a,b,c>0, a^2+b^2+c^2=1,\ \lambda\ge 0$ then:

$$\sum\frac{a^2}{1+\lambda bc}\ge\frac{3}{\lambda+3}$$

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Solution by Tapas Das-India

$$\sum a^2=1\ or\ \frac{(\sum a)^2}{3}\le 1(CBS)or\left(\sum a\right)\le\sqrt{3}\ \ (1)$$

$$abc\overset{AM-GM}{\le}.\frac{(a+b+c)^3}{27}\overset{(1)}{\le}\frac{3\sqrt{3}}{27}=\frac{\sqrt{3}}{9}\quad(2)$$

$$\sum\frac{a^2}{1+\lambda bc}=\sum\frac{a^4}{a^2+a^2\lambda bc}\overset{Bergstrom}{\ge}$$

$$\ge\frac{(a^2+b^2+c^2)^2}{a^2+b^2+c^2+\lambda abc(a+b+c)}\overset{(1),(2)\&\sum a^2=1}{\ge}.\frac{1}{1+\lambda\frac{\sqrt{3}}{9}\sqrt{3}}=\frac{3}{\lambda+3}$$

$$Equality\ for\ a=b=c=\frac{1}{\sqrt{3}}$$