ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$, $\lambda\geq 9$, $a+b+c=3$ then:

$$\sum\frac{a^3}{\lambda-a^2}\geq\frac{3}{\lambda-1}$$

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Solution by Tapas Das-India

$$\sum\frac{a^3}{\lambda-a^2}\overset{Holder}{\geq}\frac{(a+b+c)^3}{\left(3\lambda-(a^2+b^2+c^2)\right)\cdot 3}=$$

$$=\frac{(a+b+c)^3}{\left(3\cdot\left(3\lambda-(a+b+c)^2+2(ab+bc+ca)\right)\right)}\geq$$

$$\geq\frac{(a+b+c)^3}{\left(3\cdot\left(3\lambda-(a+b+c)^2+\frac{2(a+b+c)^2}{3}\right)\right)}\overset{(a+b+c=3)}{=}\frac{(3)^3}{3(3\lambda-9+6)}=\frac{3}{\lambda-1}$$

$$Equality\ for\ a=b=c=1$$