ROMANIAN MATHEMATICAL MAGAZINE

If $x, y > 0, \frac{1}{x} + \frac{1}{y} \le 2$ then:

$$\sqrt{x^3 + 8y^2} + \sqrt{y^3 + 8x^2} \ge 6$$

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$$\frac{1}{x} + \frac{1}{y} \le 2 \ or, (x + y) \le 2xy \ or$$

$$2\sqrt{xy} \le x + y \le 2xy \ (AM - GM) or \ \sqrt{xy} \ge 1 \ or \ \ xy \ge 1 \ (1)$$

$$\sqrt{x^3 + 8y^2} + \sqrt{y^3 + 8x^2} =$$

$$= \sqrt{\left(x^{\frac{3}{2}}\right)^2 + \left(2\sqrt{2}y\right)^2} + \sqrt{\left(y^{\frac{3}{2}}\right)^2 + \left(2\sqrt{2}x\right)^2} \overset{Minkowski}{\ge}$$

$$\ge \sqrt{\left(x^{\frac{3}{2}} + y^{\frac{3}{2}}\right)^2 + \left(2\sqrt{2x} + 2\sqrt{2y}\right)^2} \overset{AM-GM}{\ge}$$

$$\ge \sqrt{4(xy)^{\frac{3}{2}} + 32(xy)} \overset{(1)}{\ge} \sqrt{4 + 32} = 6$$

$$Equality \ for \ a = b = 1$$