ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ \lambda\ge 0$ then:

$$\sum\frac{\left((a+\lambda b)(c+\lambda a)\right)}{b+\lambda c}\ge(\lambda+1)(a+b+c)$$

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Solution by Tapas Das-India

$$Let\ (a+\lambda b)=x,(b+\lambda c)=y,(c+\lambda a)=z\ then:$$

$$\sum\frac{\left((a+\lambda b)(c+\lambda a)\right)}{b+\lambda c}=\sum\frac{xz}{y}=\frac{x^2z^2+x^2y^2+y^2z^2}{xyz}\ge$$

$$\ge\frac{xy\cdot yz+yz\cdot zx+zx\cdot xy}{xyz}=\frac{xyz(x+y+z)}{xyz}=x+y+z=$$

$$=(a+\lambda b)+(b+\lambda c)+(c+\lambda a)=(\lambda+1)(a+b+c)$$

$$Equality\ holds\ for\ a=b=c=1$$