ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\lambda\geq 0,n>0$ then:

$$\sum\frac{a^3}{b^2(\lambda a+nb)}\geq\frac{3}{\lambda+n}$$

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$$Vasc\ inequality:$$

$$\forall\ x,y,z>0,\frac{(x^2+y^2+z^2)^2}{3}\geq(x^3y+y^3z+z^3x)\ \ (1)\ and$$

$$3\sum a^2b^2\leq\left(\sum a^2\right)^2\ \ (2)$$

$$\sum\frac{a^3}{b^2(\lambda a+nb)}=\sum\frac{a^4}{b^2a(\lambda a+nb)}=\sum\frac{(a^2)^2}{\lambda a^2b^2+nab^3}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(a^2+b^2+c^2)^2}{\lambda(a^2\,b^2+b^2c^2+c^2a^2)+n(ab^3+bc^3+ca^3)}\overset{(1)\&(2)}{\geq}$$

$$\geq\frac{(a^2+b^2+c^2)^2}{\frac{\lambda(c^2+b^2+a^2)^2}{3}+\frac{n(a^2+b^2+c^2)^2}{3}}=\frac{3}{\lambda+n}$$

$$Equality\ holds\ for\ a=b=c=1$$