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If $a,b,c>0, abc=1$ then:

$$\sum\sqrt{1+15a^2}\ge\frac{7}{2}(a+b+c)+\frac{3}{2}$$

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Solution by Tapas Das-India

$$1+15a^2=1+(\underbrace{a^2+a^2+\cdots+a^2}_{15\ times})\ge$$

$$\overset{CBS}{\ge}\frac{\left(\left(1+(\underbrace{a+a+\cdots+a}_{15\ times}\right)\right)^2}{16}=\frac{(1+15a)^2}{16}$$

$$Now\ \sqrt{1+15a^2}\ge\frac{1+15a}{4}\ (1)$$

$$\sum\sqrt{1+15a^2}\overset{(1)}{\ge}\sum\frac{1+15a}{4}=\frac{3}{4}+\frac{15}{4}(a+b+c)=$$

$$=\frac{3}{4}+\frac{1}{4}(a+b+c)+\frac{7}{2}(a+b+c)\overset{AM-GM}{\ge}$$

$$\ge\frac{3}{4}+\frac{1}{4}3\sqrt[3]{abc}+\frac{7}{2}(a+b+c)\overset{abc=1}{=}\frac{7}{2}(a+b+c)+\frac{3}{2}$$

$$Equality\ holds\ for\ \ a=b=c=1$$