

ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, $n \in \mathbb{N}$ then:

$$\sum \frac{b+c}{a^n} \geq \frac{6(a+b+c)}{a^n + c^n + b^n}$$

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Solution by Tapas Das-India

WLOG $a \geq b \geq c$ then: $(a+b) \geq (a+c) \geq (b+c)$, $\frac{1}{a^n} \leq \frac{1}{b^n} \leq \frac{1}{c^n}$

$$\begin{aligned} \sum \frac{b+c}{a^n} &\stackrel{\text{Chebyshev}}{\geq} \frac{1}{3} \left(\sum (b+c) \right) \left(\sum \frac{1}{a^n} \right) \stackrel{\text{Bergstrom}}{\geq} \\ &\geq \frac{2}{3} (a+b+c) \frac{(1+1+1)^2}{a^n + b^n + c^n} = \frac{6(a+b+c)}{a^n + c^n + b^n} \end{aligned}$$

Equality holds for $a = b = c = 1$