ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0,\ ab+bc+ca=1$ then:

$$\sum\frac{(b+\lambda c)^2}{a^2+a(b+c)}\ge\frac{(\lambda+1)^2}{a^2+b^2+c^2}$$

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Solution by Tapas Das-India

$$\sum\frac{(b+\lambda c)^2}{a^2+a(b+c)}\overset{Bergstrom}{\ge}\frac{\left(a+b+c+\lambda(a+b+c)\right)^2}{a^2+b^2+c^2+a(b+c)+b(c+a)+c(a+b)}=$$

$$=\frac{(\lambda+1)^2(a+b+c)^2}{a^2+b^2+c^2+2ab+2bc+2ca}=\frac{(\lambda+1)^2(a+b+c)^2}{(a+b+c)^2}=$$

$$=\frac{(\lambda+1)^2}{1}=\frac{(\lambda+1)^2}{ab+bc+ca}\ (as\ \ ab+bc+ca=1)\ge\frac{(\lambda+1)^2}{a^2+b^2+c^2}$$

$$Equality\ holds\ for\ \ a=b=c=\frac{1}{\sqrt{3}}$$