ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z \geq 1$ and $\lambda \geq 0$, then :

$$\sum_{cyc} \frac{x^3 + 3x}{3(y^2 + \lambda z^2) + \lambda + 1} \geq \frac{3}{\lambda + 1}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$x^3 + 3x \overset{?}{\geq} 3x^2 + 1 \Leftrightarrow (x-1)^3 \overset{?}{\geq} 0 \to \text{true} \because x \geq 1$$
$$\therefore x^3 + 3x \geq 3x^2 + 1 \text{ and analogs}$$

$$\therefore \sum_{cyc} \frac{x^3 + 3x}{3(y^2 + \lambda z^2) + \lambda + 1} \geq \sum_{cyc} \frac{3x^2 + 1}{3y^2 + 1 + \lambda(3z^2 + 1)} =$$

$$= \sum_{cyc} \frac{a^2}{ab + \lambda ca} \ (a = 3x^2 + 1, b = 3y^2 + 1, c = 3z^2 + 1) \overset{\text{Bergstrom}}{\geq} \frac{\left(\sum_{cyc} a\right)^2}{(\lambda + 1)\left(\sum_{cyc} ab\right)} \geq$$

$$\geq \frac{\left(\sum_{cyc} a\right)^2}{(\lambda + 1)\left(\frac{\left(\sum_{cyc} a\right)^2}{3}\right)} \therefore \sum_{cyc} \frac{x^3 + 3x}{3(y^2 + \lambda z^2) + \lambda + 1} \geq \frac{3}{\lambda + 1}$$

$$\forall\, x, y, z \geq 1 \text{ and } \lambda \geq 0, '' ='' \text{ iff } x = y = z = 1 \text{ (QED)}$$