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If $a, b, c > 0$, then:

$$\frac{a^5}{a^4+b^4}+\frac{b^5}{b^4+c^4}+\frac{c^5}{c^4+a^4}+\frac{1}{2}\left(\frac{a^2}{c}+\frac{b^2}{a}+\frac{c^2}{b}\right) \geq a+b+c$$

Proposed by Marin Chirciu-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

By AM − GM inequality, we have

$$\frac{a^5}{a^4+b^4}+\frac{b^2}{2a}=a-\frac{ab^4}{a^4+b^4}+\frac{b^2}{2a} \geq a-\frac{ab^4}{2a^2b^2}+\frac{b^2}{2a}=a.$$

Similarly, we get

$$\frac{b^5}{b^4+c^4}+\frac{c^2}{2b} \geq b \quad \text{and} \quad \frac{c^5}{c^4+a^4}+\frac{a^2}{2c} \geq c.$$

Adding these inequalities yields the desired inequality.

Equality holds iff $a = b = c$.