ROMANIAN MATHEMATICAL MAGAZINE

Let $a,b,c>0$. Prove that

$$a^b+b^c+c^a>1$$

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If at least one of the numbers a,b and c is greater than or equal to 1, inequality occurs.

Assume now that $a,b,c<1$. By Bernoulli's inequality, we have

$$a^b=\frac{a}{[1+(a-1)]^{1-b}}\geq\frac{a}{1+(a-1)(1-b)}=\frac{a}{a+b-ab}>\frac{a}{a+b+c}\ \text{(and analogs)}$$

Therefore

$$a^b+b^c+c^a>\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}=1,$$

which completes the proof.