ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0, x+y+z=1$ then:

$$\sqrt{xy-2z+6}+\sqrt{yz-2x+6}+\sqrt{zx-2y+6}\leq 7$$

Proposed by Samed Ahmedov-Azerbaijan

Solution by Daniel Sitaru-Romania

$$\sum_{cyc} xy \overset{GM-QM}{\leq} \sum_{cyc}\frac{x^2+y^2}{2}=\sum_{cyc} x^2=(x+y+z)^2-2\sum_{cyc} xy=$$

$$=1-2\sum_{cyc} xy\Rightarrow 3\sum_{cyc} xy\leq 1\Rightarrow$$

$$\sum_{cyc} xy\leq \frac{1}{3}\quad (1)$$

$$\sum_{cyc}\sqrt{xy-2z+6}\overset{CBS}{\leq}\sqrt{3\sum_{cyc}(xy-2z+6)}\leq$$

$$\leq\sqrt{3\sum_{cyc} xy-6\sum_{cyc} x+54}\overset{(1)}{\leq}\sqrt{3\cdot\frac{1}{3}-6\cdot 1+54}=\sqrt{49}=7$$

Equality holds for $x=y=z=\frac{1}{3}$.