ROMANIAN MATHEMATICAL MAGAZINE

Let $x, y, z \geq 0$ such that $x+y+z=3$. Prove that

$$\frac{1}{2x^2+2}+\frac{1}{2y^2+2}+\frac{1}{2z^2+2} \geq \frac{3}{4}$$

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For $x > 0$, we have

$$\frac{1}{2x^2+2}=\frac{1}{2}-\frac{x^2}{2(x^2+1)} \overset{AM-GM}{\geq} \frac{1}{2}-\frac{x^2}{2.2x}=\frac{1}{2}-\frac{x}{4},$$

which is also true for $x=0$. Then

$$\frac{1}{2x^2+2}+\frac{1}{2y^2+2}+\frac{1}{2z^2+2} \geq \left(\frac{1}{2}-\frac{x}{4}\right)+\left(\frac{1}{2}-\frac{y}{4}\right)+\left(\frac{1}{2}-\frac{z}{4}\right)=\frac{3}{2}-\frac{x+y+z}{4}=$$

$$=\frac{3}{2}-\frac{3}{4}=\frac{3}{4}$$

Equality holds for: $x=y=z=1$.