ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0, x + y + z = 1$ then:

$$\sqrt{x + yz} + \sqrt{y + xz} + \sqrt{z + xy} \le 2$$

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$$\sqrt{x + yz} + \sqrt{y + xz} + \sqrt{z + xy} =$$

$$= \sqrt{(1 - y - z) + zy} + \sqrt{(1 - x - z) + xz} + \sqrt{(1 - y - x) + xy} =$$

$$= \sqrt{(1 - y)(1 - z)} + \sqrt{(1 - x)(1 - z)} + \sqrt{(1 - x)(1 - y)} \le$$

$$\overset{AM-GM}{\le} \frac{(1 - y) + (1 - z)}{2} + \frac{(1 - x) + (1 - z)}{2} + \frac{(1 - x) + (1 - y)}{2} =$$

$$= \frac{6 - 2(x + y + z)}{2} = 2$$

Equality holds for $x = y = z = \frac{1}{3}$.