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If $a,b,c>0, abc=1, n\in\mathbb{N}$ then:

$$\frac{a^{n+2}+b^{n+2}}{c^n(a^{n+1}+b^{n+1})}+\frac{b^{n+2}+c^{n+2}}{a^n(b^{n+1}+c^{n+1})}+\frac{c^{n+2}+a^{n+2}}{b^n(c^{n+1}+a^{n+1})}\ge 3$$

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Solution by Daniel Sitaru-Romania

$$\sum_{cyc}\frac{a^{n+2}+b^{n+2}}{c^n(a^{n+1}+b^{n+1})}\overset{LEHMER}{\geq}\sum_{cyc}\frac{a+b}{2c^n}\overset{AM-GM}{\geq}\sum_{cyc}\frac{\sqrt{ab}}{c^n}\geq$$

$$\overset{AM-GM}{\geq} 3\sqrt[3]{\frac{\sqrt{ab}}{c^n}\cdot\frac{\sqrt{bc}}{a^n}\cdot\frac{\sqrt{ca}}{b^n}}=3\sqrt[3]{\frac{abc}{(abc)^n}}=3\sqrt[3]{1}=3$$

Equality holds for $a=b=c=1$.