ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1$ then:

$$\frac{b(b+c)^3+a(a+c)^3}{c(a+b)}+\frac{b(a+b)^3+c(a+c)^3}{a(b+c)}+\frac{a(a+b)^3+c(b+c)^3}{b(a+c)}\geq 24$$

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Solution by Daniel Sitaru-Romania

$$\frac{b(b+c)^3+a(a+c)^3}{c(a+b)}+\frac{b(a+b)^3+c(a+c)^3}{a(b+c)}+\frac{a(a+b)^3+c(b+c)^3}{b(a+c)}=$$

$$=\frac{b(b+c)^3}{c(a+b)}+\frac{a(a+c)^3}{c(a+b)}+\frac{b(a+b)^3}{a(b+c)}+\frac{c(a+c)^3}{a(b+c)}+\frac{a(a+b)^3}{b(a+c)}+\frac{c(b+c)^3}{b(a+c)}\overset{AM-GM}{\geq}$$

$$6\cdot\sqrt[6]{\frac{b(b+c)^3}{c(a+b)}\cdot\frac{a(a+c)^3}{c(a+b)}\cdot\frac{b(a+b)^3}{a(b+c)}\cdot\frac{c(a+c)^3}{a(b+c)}\cdot\frac{a(a+b)^3}{b(a+c)}\cdot\frac{c(b+c)^3}{b(a+c)}}=$$

$$=6\cdot\sqrt[6]{\left(\prod_{cyc}(a+b)\right)^4}=6\cdot\sqrt[3]{\left(\prod_{cyc}(a+b)\right)^2}\overset{AM-GM}{\geq}6\cdot\sqrt[3]{\left(\prod_{cyc}(2\sqrt{ab})\right)^2}=$$

$$=6\cdot\sqrt[3]{64(abc)^2}=6\cdot\sqrt[3]{64}=6\cdot 4=24$$

Equality holds for: $a=b=c=1$.