ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1$ then:

$$(a^{2024}+b^{2024})^{2025}+(b^{2024}+c^{2024})^{2025}+(c^{2024}+a^{2024})^{2025}\ge 3\cdot 2^{2025}$$

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Solution by Daniel Sitaru-Romania

$$\sum_{cyc}(a^{2024}+b^{2024})^{2025}\overset{AM-GM}{\ge}\sum_{cyc}\left(2\sqrt{a^{2024}\cdot b^{2024}}\right)^{2025}=$$

$$=2^{2025}\cdot\sum_{cyc}(a^{1012}\cdot b^{1012})^{2025}\overset{AM-GM}{\ge}2^{2025}\cdot 3\sqrt[3]{\left(\prod_{cyc}a^{2024}\right)^{2025}}=$$

$$=3\cdot 2^{2025}\cdot(abc)^{\frac{1}{3}\cdot 2024\cdot 2025}=3\cdot 2^{2025}\cdot(1)^{\frac{1}{3}\cdot 2024\cdot 2025}=3\cdot 2^{2025}$$

Equality holds for $a=b=c=1$.