ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $abc=1$, then prove that

$$\frac{(ab)^{2024}\left(b^{2024}c^2+a^{2026}\right)}{c^2\left(a^{2025}+b^{2025}\right)}+\frac{(bc)^{2024}\left(c^{2024}a^2+b^{2026}\right)}{a^2\left(b^{2025}+c^{2025}\right)}+\frac{(ca)^{2024}\left(a^{2024}b^2+c^{2026}\right)}{b^2\left(c^{2025}+a^{2025}\right)}\geq 3$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

Let $x=a^{2024}b^{2026},\ \ y=b^{2024}c^{2026},\ \ z=c^{2024}a^{2026}$. We have

$$\sum_{cyc}\frac{(ab)^{2024}(b^{2024}c^2+a^{2026})}{c^2(a^{2025}+b^{2025})}=\sum_{cyc}\frac{(ab)^{2025}.\,c^{2024}(b^{2024}c^2+a^{2026})}{c^{2025}(a^{2025}+b^{2025})}=$$

$$=\sum_{cyc}\frac{(ab)^{2025}.\,(y+z)}{(bc)^{2025}+(ca)^{2025}}=\sum_{cyc}(ab)^{2025}.\sum_{cyc}\frac{y+z}{(bc)^{2025}+(ca)^{2025}}-2\sum_{cyc}x\geq$$

$$\overset{CBS}{\geq}\sum_{cyc}(ab)^{2025}.\frac{\left(\sum_{cyc}\sqrt{y+z}\right)^2}{2\sum_{cyc}(ab)^{2025}}-2\sum_{cyc}x=\sum_{cyc}\left(\sqrt{(x+y)(x+z)}-x\right)\geq$$

$$\overset{CBS}{\geq}\sum_{cyc}\left(x+\sqrt{yz}-x\right)=\sum_{cyc}\sqrt{yz}\overset{AM-GM}{\geq}3\sqrt[3]{xyz}=3(abc)^{1350}=3,$$

as desired. Equality holds iff $a=b=c=1$.