ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$ then:

$$\sum \frac{b^2\left(\frac{a+c}{a+b}\right)^2 + c^2\left(\frac{b+c}{a+b}\right)^2}{(a+c)^2 + (b+c)^2} \geq \frac{9(abc)^{\frac{2}{3}}}{4(a^2+b^2+c^2)}$$

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Solution by Tapas Das-India

$$(a+c)^2 + (b+c)^2 \overset{CBS}{\leq} 2(a^2+c^2) + 2(b^2+c^2) = 2(a^2+b^2+2c^2) \quad (1)$$

$$b^2\left(\frac{a+c}{a+b}\right)^2 + c^2\left(\frac{b+c}{a+b}\right)^2 \overset{AM-GM}{\geq} \frac{2bc(a+c)(b+c)}{(a+b)^2} \quad (2)$$

$$\sum \frac{b^2\left(\frac{a+c}{a+b}\right)^2 + c^2\left(\frac{b+c}{a+b}\right)^2}{(a+c)^2 + (b+c)^2} \overset{(1)\&(2)}{\geq} \sum \frac{\frac{2bc(a+c)(b+c)}{(a+b)^2}}{2(a^2+b^2+2c^2)} \overset{AM-GM}{\geq}$$

$$\geq 3\sqrt[3]{\frac{a^2b^2c^2}{\prod a^2+b^2+2c^2}} \overset{AM-GM}{\geq} \frac{3(abc)^{\frac{2}{3}}}{\frac{\sum(a^2+b^2+2c^2)}{3}} = \frac{9(abc)^{\frac{2}{3}}}{4(a^2+b^2+c^2)}$$

$$Equality\ holds\ \ for\ a = b = c$$