ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, abc=1$ then:

$$\frac{a^6+b^6}{c^4(a^5+b^5)}+\frac{b^6+c^6}{a^4(b^5+c^5)}+\frac{c^6+a^6}{b^4(c^5+a^5)}\geq 3$$

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Solution by Daniel Sitaru-Romania

$$\frac{a^6+b^6}{c^4(a^5+b^5)}+\frac{b^6+c^6}{a^4(b^5+c^5)}+\frac{c^6+a^6}{b^4(c^5+a^5)}=\sum_{cyc}\frac{a^6+b^6}{c^4(a^5+b^5)}\geq$$

$$\overset{LEHMER}{\geq}\sum_{cyc}\frac{a^5+b^5}{c^4(a^4+b^4)}\overset{LEHMER}{\geq}\sum_{cyc}\frac{a^4+b^4}{c^4(a^3+b^3)}\overset{LEHMER}{\geq}\sum_{cyc}\frac{a^3+b^3}{c^4(a^2+b^2)}\geq$$

$$\overset{LEHMER}{\geq}\sum_{cyc}\frac{a^2+b^2}{c^4(a^1+b^1)}\overset{LEHMER}{\geq}\sum_{cyc}\frac{a^1+b^1}{c^4(a^0+b^0)}\geq\frac{1}{2}\sum_{cyc}\left(\frac{a}{c^4}+\frac{b}{c^4}\right)\overset{AM-GM}{\geq}$$

$$\geq\frac{1}{2}\cdot 6\sqrt[6]{\frac{a}{c^4}\cdot\frac{b}{c^4}\cdot\frac{b}{a^4}\cdot\frac{c}{a^4}\cdot\frac{c}{b^4}\cdot\frac{a}{b^4}}=3\sqrt[6]{\frac{1}{(abc)^6}}=3\sqrt[6]{\frac{1}{1^6}}=3$$

Equality holds for $a=b=c=1$.