ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0$, then prove that

$$\frac{a^4b^4(a^6+b^6)}{a^5+b^5}+\frac{b^4c^4(b^6+c^6)}{b^5+c^5}+\frac{c^4a^4(c^6+a^6)}{c^5+a^5}\ge\frac{1}{9}a^2b^2c^2(a+b+c)^3$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$\sum_{cyc}\frac{a^4b^4(a^6+b^6)}{a^5+b^5}\overset{Chebyshev}{\ge}\sum_{cyc}\frac{a^4b^4(a^5+b^5)(a+b)}{2(a^5+b^5)}=\sum_{cyc}\frac{a^4b^4(a+b)}{2}$$

$$=\frac{1}{2}\sum_{cyc}a^5(b^4+c^4)\overset{AM-GM}{\ge}\frac{1}{2}\sum_{cyc}a^5.2b^2c^2=$$

$$=\frac{1}{9}a^2b^2c^2.3^2\sum_{cyc}a^3\overset{Hölder}{\ge}\frac{1}{9}a^2b^2c^2(a+b+c)^3$$

Equality holds iff $a=b=c$.