If $a,b,c,\alpha\geq 1$ then:

$$\frac{a}{c}+\frac{b}{a}+\frac{c}{b}+\alpha(ab+bc+ca)\geq(1+\alpha)(a+b+c)$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Ravi Prakash-India

For $x,y,\alpha\geq 1$, $x\left(\alpha-\frac{1}{y}\right)(y-1)\geq 0\Rightarrow \alpha xy+\frac{x}{y}-x-\alpha x\geq 0$

$$\Rightarrow\frac{x}{y}+\alpha xy\geq(1+\alpha)x$$

$$\therefore$$

$$\frac{a}{c}+\alpha ac\geq(1+\alpha)a$$

$$\frac{b}{a}+\alpha ab\geq(1+\alpha)b$$

and $\frac{c}{b}+\alpha bc\geq(1+\alpha)c$

Adding above inequalities the desired inequality.

Solution 2 by Tapas Das-India

We need to show,

$$\frac{a}{c}+\frac{b}{a}+\frac{c}{b}+\alpha(ab+bc+ca)\geq(1+\alpha)(a+b+c)$$

$$\left[\frac{a}{c}+\alpha ac-(1+\alpha)a\right]+\left[\frac{b}{a}+\alpha ab-(a+\alpha)b\right]+\left[\frac{c}{b}+\alpha bc-(1+\alpha)c\right]\geq 0$$

or, $a\left[\frac{1}{c}+\alpha c-(1+\alpha)\right]+b\left[\frac{1}{a}+\alpha a-(1+\alpha)\right]+c\left[\frac{1}{b}+\alpha b-(1+\alpha)\right]\geq 0$

or, $a\left[\frac{(1-c)(1-c\alpha)}{c}\right]+b\left[\frac{(1-a)(1-a\alpha)}{a}\right]+c\left[\frac{(1-b)(1-b\alpha)}{b}\right]\geq 0$

or, $\frac{a}{c}(1-c)(1-c\alpha)+\frac{b}{a}(1-a)(1-a\alpha)+\frac{c}{b}(1-b)(1-b\alpha)\geq 0$

This is true. Since, $c\geq 1,\alpha\geq 1\therefore c\alpha\geq 1$

$$\therefore(1-c)\leq 0,(1-c\alpha)\leq 0$$

$\therefore(1-c)(1-c\alpha)\geq 0$ (analog)