

ROMANIAN MATHEMATICAL MAGAZINE

If $x \in (0, \frac{\pi}{2})$ then:

$$2(\sin^2 x + \csc^2 x)^2 + 2(\cos^2 x + \sec^2 x)^2 \geq 25$$

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$$\begin{aligned}x &\in \left(0, \frac{\pi}{2}\right) \\2\left(\sin^2(x) + \frac{1}{\sin^2(x)}\right)^2 + 2\left(\cos^2(x) + \frac{1}{\cos^2(x)}\right)^2 &\geq 25? * a = \sin^2(x), b = \cos^2(x), a + b = 1 \\ \Leftrightarrow 2\left(a + \frac{1}{a}\right)^2 + 2\left(b + \frac{1}{b}\right)^2 &\geq 2 \cdot \frac{1}{2} \left(a + \frac{1}{a} + b + \frac{1}{b}\right)^2 \\= \left(1 + \frac{a+b}{ab}\right)^2 &\geq \left(1 + \frac{1}{\frac{(a+b)^2}{4}}\right)^2 = 25 \\ \text{Equality holds iff } a = b = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{4} &\end{aligned}$$