If $x \in \left(0, \frac{\pi}{2}\right)$ then:

$$2(\sin^2 x + \csc^2 x)^2 + 2(\cos^2 x + \sec^2 x)^2 \geq 25$$

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Solution by Pham Duc Nam-Vietnam

$$x \in \left(0, \frac{\pi}{2}\right)$$

$$2\left(\sin^2(x) + \frac{1}{\sin^2(x)}\right)^2 + 2\left(\cos^2(x) + \frac{1}{\cos^2(x)}\right)^2 \geq 25?$$

$$* \, a = \sin^2(x), b = \cos^2(x), a + b = 1$$

$$\Leftrightarrow 2\left(a + \frac{1}{a}\right)^2 + 2\left(b + \frac{1}{b}\right)^2 \geq 2 \cdot \frac{1}{2}\left(a + \frac{1}{a} + b + \frac{1}{b}\right)^2$$

$$= \left(1 + \frac{a+b}{ab}\right)^2 \geq \left(1 + \frac{1}{\frac{(a+b)^2}{4}}\right)^2 = 25$$

Equality holds iff $a = b = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{4}$