If $a, b \in \mathbb{C}, n \in \mathbb{N}^*$ then:

$$|a+b-2-2i|^n \le 2^{n-1}(|a-1-i|^n + |b-1-i|^n)$$

Proposed by Daniel Sitaru – Romania

Solution by Hikmat Mammadov-Azerbaijan

$$|a+b-2-2i|^n \le 2^{n-1}(|a-1-i|^n + |b-1-i|^n)$$

We will use Clarkson's Inequality

If we let

$z = a-(1+i)$ **and** $w = b-(1+i)$

Then

$$|a+b-2-2i|^n = |z+w|^n \le |z+w|^n + |z-w|^n \le$$

$$\le 2^{n-1}(|z|^n + |w|^n) = 2^{n-1}(|a-1-i|^n + |b-1-i|^n)$$

Hence

$$|a+b-2-2i|^n \le 2^{n-1}(|a-1-i|^n + |b-1-i|^n)$$

Note

The second-to-last inequality follows Clarkson's inequality