ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z \geq 0$ then:

$$\sqrt{x^2+z^2+xz}+\sqrt{y^2+z^2+yz\sqrt{3}} \geq \sqrt{x^2+y^2}$$

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Solution 1 by Ravi Prakash-New Delhi-India, Solution 2 by proposer

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Let be $\omega \in \mathbb{C}, \omega^3 = 1, \omega \neq 1.$

$$x^2+z^2+xz = |x+\omega z|^2, \qquad y^2+z^2+yz\sqrt{3} = |iy-\omega z|^2$$

$$\sqrt{x^2+z^2+xz}+\sqrt{y^2+z^2+yz\sqrt{3}} = |x+\omega z|+|iy-\omega z| \geq$$

$$\geq |x+\omega z+iy-\omega z| = |x+iy| = \sqrt{x^2+y^2}$$

Equality holds for $x = y = z = 0$.

Solution 2 by proposer

Let be $M \in Int(\Delta ABC)$ such that:

$$m(\sphericalangle AMB) = 90^\circ, m(\sphericalangle BMC) = 150^\circ, m(\sphericalangle CMA) = 120^\circ$$

$$\begin{cases} AC^2 = x^2+z^2-2xz\cos 120^\circ = x^2+z^2+xz \\ BC^2 = y^2+z^2-2yz\cos 150^\circ = y^2+z^2+yz\sqrt{3} \Rightarrow \\ AB^2 = x^2+y^2 \end{cases}$$

$$\Rightarrow \begin{cases} AC = \sqrt{x^2+z^2+xz} \\ BC = \sqrt{y^2+z^2+yz\sqrt{3}}, \\ AB = \sqrt{x^2+y^2} \end{cases} \qquad AC + BC \geq AB \Rightarrow$$

$$\sqrt{x^2+z^2+xz}+\sqrt{y^2+z^2+yz\sqrt{3}} \geq \sqrt{x^2+y^2}$$

Equality holds for $x = y = z = 0$.