ROMANIAN MATHEMATICAL MAGAZINE

If $0 \leq x, y, z \leq \frac{\pi}{4}, x + y + z = \frac{\pi}{4}$ then:

$$1 + \tan x \cdot \tan y \cdot \tan z > 4\sqrt{\tan x \cdot \tan y \cdot \tan z}$$

Proposed by Daniel Sitaru – Romania

Solution by Tapas Das – India

$$x + y + z = \frac{\pi}{4}$$

$$\therefore \tan(x + y + z) = 1$$

$$\frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x} = 1$$

$$\therefore 1 + \tan x \tan y \tan z$$

$$= \tan x + \tan y + \tan z + \tan x \tan y + \tan y \tan z + \tan z \tan x$$

$$\overset{AM-GM}{\geq} 3(\tan x \tan y \tan z)^{\frac{1}{3}} + 3(\tan x \tan y \tan z)^{\frac{2}{3}}$$

$$\overset{AM-GM}{\geq} 6 \cdot \sqrt{(\tan x \tan y \tan z)^{\frac{1}{3}} \cdot (\tan x \tan y \tan z)^{\frac{2}{3}}} = 6\sqrt{\tan x \tan y \tan z}$$

$$> 4\sqrt{\tan x \tan y \tan z}$$