ROMANIAN MATHEMATICAL MAGAZINE

If $x \in \left(0, \frac{\pi}{4}\right)$ then
$$2\sin x + \tan x < 3x + \ln x \cdot \ln(1-x)$$

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Using the well known inequality, $\ln t \le t - 1, \forall t > 0$,
we have $-\ln x > 1 - x > 0$ and,
$-\ln(1-x) > x > 0$, then $\ln x . \ln(1-x) > (1-x)x$, $\forall x \in \left(0, \frac{\pi}{4}\right)$.
So it suffices to prove that $f(x) = 4x - x^2 - 2\sin x - \tan x \ge 0$, $\forall x \in \left[0, \frac{\pi}{4}\right]$.
We have
$f''(x) = -2(1 - \sin x) - 2(1 + \tan^2 x)\tan x < 0$, then f is concave on $\left[0, \frac{\pi}{4}\right]$, and
$$f(x) \ge \min\left\{f(0), f\left(\frac{\pi}{4}\right)\right\} = \min\left\{0, \pi - \frac{\pi^2}{16} - \sqrt{2} - 1\right\} = 0, \forall x \in \left[0, \frac{\pi}{4}\right],$$
which completes the proof.