ROMANIAN MATHEMATICAL MAGAZINE

If $a, \mathrm{b}, \mathrm{c} > 0$, *then* :

$$a^4 + b^4 + c^4 + 19(a^2 + b^2 + c^2) > 28(a + b + c) + 18abc - 48$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} a^4 + 19\sum_{cyc} a^2 - 28\sum_{cyc} a + 48 - 18abc \overset{\substack{\text{Holder}\\+\\\text{A-G}}}{\geq}$$

$$\frac{1}{27}\cdot\left(\sum_{cyc} a\right)^4 + \frac{19}{3}\cdot\left(\sum_{cyc} a\right)^2 - 28\sum_{cyc} a + 48 - \frac{18}{27}\cdot\left(\sum_{cyc} a\right)^3 \overset{?}{>} 0$$

$$\Leftrightarrow t^4 - 18t^3 + 171t^2 - 756t + 1296 \overset{?}{>} 0 \left(t = \sum_{cyc} a\right)$$

$$\Leftrightarrow (t-4)^2(t^2 - 10t + 75) + 4t + 96 \overset{?}{>} 0$$

$$\Leftrightarrow (t-4)^2\left((t-5)^2 + 50\right) + 4t + 96 \overset{?}{>} 0 \rightarrow \text{true} \because t = \sum_{cyc} a > 0$$

$$\therefore a^4 + b^4 + c^4 + 19(a^2 + b^2 + c^2) > 28(a + b + c) + 18abc - 48 \text{ (QED)}$$