ROMANIAN MATHEMATICAL MAGAZINE

Compare:

$$\sqrt{2024+\sqrt{2024-\sqrt{2023+\sqrt{2022}}}} \text{ and } \pi^{\sqrt{e}}$$

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Since $4>\pi$ and $4>e$, we have $45>16>\pi^{\sqrt{e}}$, so it is sufficient to prove that the radical expression is greater than 45.

If we denote $x=\sqrt{2024-\sqrt{2023+\sqrt{2022}}}$, then we have

$$\sqrt{2024+x}>45 \Leftrightarrow 2024+x>2025 \Leftrightarrow x>1 \Leftrightarrow 2024-\sqrt{2023+\sqrt{2022}}>1 \Leftrightarrow$$

$$\Leftrightarrow 2023>\sqrt{2023+\sqrt{2022}} \Leftrightarrow 2023\cdot 2022>\sqrt{2022} \Leftrightarrow 2023\cdot\sqrt{2022}>0$$

In conclusion, we have $\sqrt{2024+\sqrt{2024-\sqrt{2023+\sqrt{2022}}}}>\pi^{\sqrt{e}}$