ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0$ prove that

$$\left(\frac{2b}{b+2a}+\sqrt[3]{\left(\frac{b}{a}\right)^2}+\frac{4(a^2+1)}{3(1+ab)}\right)\cdot\sqrt[9]{\frac{a^2(b+2a)^2(9+2a^2+2b^2+5ab)^4}{b^4(a^2+1)^4}}\geq 9\sqrt[9]{3}$$

Proposed by Pavlos Trifon-Greece

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

By AM − GM inequality, we have

$$\frac{2b}{b+2a}+\sqrt[3]{\left(\frac{b}{a}\right)^2}+\frac{4(a^2+1)}{3(1+ab)}=2.\frac{b}{b+2a}+3.\frac{1}{3}\sqrt[3]{\left(\frac{b}{a}\right)^2}+4.\frac{a^2+1}{3(1+ab)}$$

$$\geq 9\sqrt[9]{\left(\frac{b}{b+2a}\right)^2\cdot\left(\frac{1}{3}\sqrt[3]{\left(\frac{b}{a}\right)^2}\right)^3\cdot\left(\frac{a^2+1}{3(1+ab)}\right)^4}$$

$$=9\sqrt[9]{\frac{3b^4(a^2+1)^4}{a^2(b+2a)^2(9+9ab)^4}}$$

$$=9\sqrt[9]{\frac{3b^4(a^2+1)^4}{a^2(b+2a)^2(9+4ab+5ab)^4}}\geq 9\sqrt[9]{\frac{3b^4(a^2+1)^4}{a^2(b+2a)^2(9+2a^2+2b^2+5ab)^4}},$$

$$\Rightarrow\left(\frac{2b}{b+2a}+\sqrt[3]{\left(\frac{b}{a}\right)^2}+\frac{4(a^2+1)}{3(1+ab)}\right)\cdot\sqrt[9]{\frac{a^2(b+2a)^2(9+2a^2+2b^2+5ab)^4}{b^4(a^2+1)^4}}\geq 9\sqrt[9]{3}.$$

Equality holds iff $a = b$.