ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$(1+x)\sqrt{1-x}+(1-x)\sqrt{1+x}=2\sqrt{1+x^2}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$(1+x)\sqrt{1-x}+(1-x)\sqrt{1+x}=2\sqrt{1+x^2}$$

$$\sqrt{1-x^2}\left(\sqrt{1+x}+\sqrt{1-x}\right)=2\sqrt{1+x^2}\ ,-1\le x\le 1$$

$$(1-x^2)\left(2+2\sqrt{1-x^2}\right)=4(1+x^2)$$

$$Let\ \ x=\sin(t)$$

$$1.\ \left(1-sin^2(t)\right)(1+\cos(t))=2\left(2-cos^2(t)\right)$$

$$case\ \ \cos(t)\ge 0$$

$$cos^3(t)+3cos^2(t)-4=0,\qquad (\cos(t)-1)(\cos(t)+2)^2=0$$

$$\cos(t)=1\ ;\ \cos(t)\ne 2$$

$$\cos(t)=1\to\ \ x=0$$

$$2.\ \left(1-sin^2(t)\right)(1-\cos(t))=2\left(2-cos^2(t)\right)$$

$$case\ \ \cos(t)<0$$

$$cos^3(t)-3cos^2(t)+4=0,\qquad (\cos(t)+1)(\cos(t)-2)^2=0$$

$$\cos(t)=-1\ ;\ \cos(t)\ne 2$$

$$\cos(t)=-1\to\ \ x=0$$

$$So\ \ answer\ \{0\}$$