

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt{x-y} + 3\sqrt{y-z} + 5\sqrt{z+x} = x + \frac{35}{2}$$

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Change of variable: $\sqrt{x-y} = a$; $\sqrt{y-z} = b$; $\sqrt{z+x} = c$
 $x-y = a^2$; $y-z = b^2$; $z+x = c^2 \Rightarrow a^2 + b^2 + c^2 = 2x$

Then we have:

$$a + 3b + 5c = x + \frac{35}{2} \Rightarrow 2a + 6b + 10c - 35 = 2x$$

The equality is:

$$\begin{aligned} a^2 + b^2 + c^2 &= 2a + 6b + 10c - 35 \\ a^2 - 2a + 1 + b^2 - 6b + 9 + c^2 - 10c + 25 &= 0 \\ (a-1)^2 + (b-3)^2 + (c-5)^2 &= 0 \Leftrightarrow \\ (a-1)^2 = 0 \wedge (b-3)^2 = 0 \wedge (c-5)^2 = 0 &\Leftrightarrow \end{aligned}$$

Then we have:

$$\begin{aligned} a = 1 \wedge b = 3 \wedge c = 5 &\Rightarrow \\ \sqrt{x-y} = 1; \sqrt{y-z} = 3; \sqrt{z+x} = 5 &\Rightarrow \\ x-y = 1; y-z = 9; z+x = 25 &\Rightarrow \end{aligned}$$

Solving the system:

$$\text{Answer: } x = \frac{35}{2}; y = \frac{33}{2}; z = \frac{15}{2}$$