

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$x^2 - 2x + 29 = 2\sqrt{x^2 - x + 1} + 4\sqrt{x + 3} + 6\sqrt{11 - 2x}$$

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Change of variables:

$$\sqrt{x^2 - x + 1} = a \rightarrow x^2 - x + 1 = a^2$$

$$\sqrt{x + 3} = b \rightarrow x + 3 = b^2$$

$$\sqrt{11 - 2x} = c \rightarrow 11 - 2x = c^2$$

Sum of values: $x^2 - 2x + 15 = a^2 + b^2 + c^2$

$$x^2 - 2x + 15 + 14 = a^2 + b^2 + c^2 + 14$$

$$x^2 - 2x + 15 + 29 = a^2 + b^2 + c^2 + 14$$

Replace the values: $a^2 + b^2 + c^2 + 14 = 2a + 4b + 6c$

$$a^2 - 2a + 1 + b^2 - 4b + 4 + c^2 - 6c + 9 = 0$$

$$(a - 1)^2 + (b - 2)^2 + (c - 3)^2 = 0 \rightarrow$$

$$a - 1 = 0 \vee b - 2 = 0 \vee c - 3 = 0$$

$$a = 1 \vee b = 2 \vee c = 3$$

But we know: $\sqrt{x^2 - x + 1} = 1 \vee \sqrt{x + 3} = 2 \vee \sqrt{11 - 2x} = 3$

$$x^2 - x + 1 = 1 \vee x + 3 = 4 \vee 11 - 2x = 9$$

$$x(x - 1) = 0 \vee x = 1 \vee 2 = 2x$$

$$x = 0 \vee x = 1 \vee x = 1 \vee x = 1. \text{ Answer: } x = 1$$