ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a^2+m_b^2-c^2}{sin^2(A)+sin^2(B)}+\frac{m_a^2+m_c^2-b^2}{sin^2(A)+sin^2(C)}+\frac{m_b^2+m_c^2-a^2}{sin^2(B)+sin^2(C)}=3R^2$$

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$$\sin(A)=\frac{a}{2R}\ ;\ \sin(B)=\frac{b}{2R}\ ;\ \sin(C)=\frac{c}{2R}\quad(1)$$

$$Also\rightarrow\begin{cases}4m_a^2=2(b^2+c^2)-a^2\\4m_b^2=2(a^2+c^2)-b^2\\4m_c^2=2(b^2+a^2)-c^2\end{cases}(2)$$

$$Let's\ write\ the\ expression\ (1)\ instead$$

$$R^2\left(\frac{4(m_a^2+m_b^2-c^2)}{b^2+a^2}+\frac{4(m_a^2+m_c^2-b^2)}{c^2+a^2}+\frac{4(m_c^2+m_b^2-a^2)}{b^2+c^2}\right)=$$

$$R^2\left(\frac{4m_a^2+4m_b^2-4c^2}{b^2+a^2}+\frac{4m_a^2+4m_c^2-4b^2}{c^2+a^2}+\frac{4m_c^2+4m_b^2-4a^2}{b^2+c^2}\right)\overset{(2)}{=}$$

$$R^2\Big(\frac{2(b^2+c^2)-a^2+2(a^2+c^2)-b^2-4c^2}{b^2+a^2}+\frac{2(b^2+c^2)-a^2+2(a^2+b^2)-c^2-4b^2}{c^2+a^2}+$$

$$+\frac{2(a^2+c^2)-b^2+2(a^2+b^2)-c^2-4a^2}{b^2+c^2}\Big)=$$

$$R^2\left(\frac{b^2+a^2}{b^2+a^2}+\frac{c^2+a^2}{c^2+a^2}+\frac{b^2+c^2}{b^2+c^2}\right)=3R^2\ (proved)$$