

ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{4m_b^2 - b^2}{4m_a^2 - a^2} = \sum_{cyc} \frac{\tan A}{\tan B}$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \sum_{cyc} \frac{4m_b^2 - b^2}{4m_a^2 - a^2} &= \sum_{cyc} \frac{2(a^2 + c^2) - b^2 - b^2}{2(b^2 + c^2) - a^2 - a^2} = \\ &= \sum_{cyc} \frac{a^2 + c^2 - b^2}{b^2 + c^2 - a^2} = \sum_{cyc} \frac{2a \cos B}{2b \cos A} = \sum_{cyc} \frac{a \cos B}{b \cos A} = \\ &= \sum_{cyc} \frac{2R \sin A \cos B}{2R \sin B \cos A} = \sum_{cyc} \frac{\frac{\sin A}{\cos A}}{\frac{\sin B}{\cos B}} = \sum_{cyc} \frac{\tan A}{\tan B} \end{aligned}$$