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In $\triangle ABC$, I_a, I_b, I_c – excenters, the following relationship holds:

$$\frac{I_b I_c}{w_a} + \frac{I_c I_a}{w_b} + \frac{I_a I_b}{w_c} = \frac{2R}{F} \cdot (h_a + h_b + h_c)$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \sum_{cyc} \frac{I_b I_c}{w_a} &= \sum_{cyc} \frac{4R \cos \frac{A}{2}}{\frac{2bc}{b+c} \cos \frac{A}{2}} = 2R \sum_{cyc} \frac{b+c}{bc} = 2R \sum_{cyc} \left(\frac{1}{b} + \frac{1}{c} \right) = \\ &= 4R \sum_{cyc} \frac{1}{a} = \frac{2R}{F} \sum_{cyc} \frac{2F}{a} = \frac{2R}{F} \cdot (h_a + h_b + h_c) \end{aligned}$$