



Suppose that

$$\angle BAD = 20^\circ$$

$$\angle DBA = 40^\circ$$

$$\angle DAC = 20^\circ$$

Prove that

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{b}$$

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$$m(\sphericalangle ABD) = 180^\circ - 40^\circ - 20^\circ = 120^\circ, m(\sphericalangle ADC) = 180^\circ - 120^\circ = 60^\circ$$

In $\triangle ADC$:

$$\frac{a}{\sin 100^\circ} = \frac{b}{\sin 20^\circ} \Rightarrow b = \frac{a \sin 20^\circ}{\sin 100^\circ}$$

In $\triangle ADB$:

$$\frac{c}{\sin 20^\circ} = \frac{a}{\sin 40^\circ} \Rightarrow c = \frac{a \sin 20^\circ}{\sin 40^\circ}$$

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{b} \Leftrightarrow \frac{1}{a} + \frac{1}{\frac{a \sin 20^\circ}{\sin 40^\circ}} = \frac{1}{\frac{a \sin 20^\circ}{\sin 100^\circ}} \Leftrightarrow \sin 20^\circ + \sin 40^\circ = \sin 100^\circ \Leftrightarrow$$

$$\Leftrightarrow 2 \sin \frac{20^\circ + 40^\circ}{2} \cos \frac{20^\circ - 40^\circ}{2} = \sin(90^\circ + 10^\circ) \Leftrightarrow$$

$$\Leftrightarrow 2 \sin 30^\circ \cos 10^\circ = \sin 90^\circ \cos 10^\circ + \sin 10^\circ \cos 90^\circ \Leftrightarrow$$

$$\Leftrightarrow 2 \cdot \frac{1}{2} \cos 10^\circ = 1 \cdot \cos 10^\circ + \sin 10^\circ \cdot 0 \Leftrightarrow \cos 10^\circ = \cos 10^\circ$$