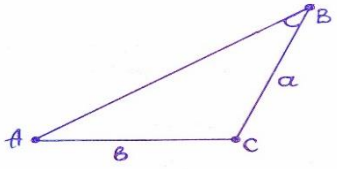
ROMANIAN MATHEMATICAL MAGAZINE

Suppose that ∠ABC=20^0, ∠CAB=40^0. Prove that: $a^3 - b^3 = 3ab^2$

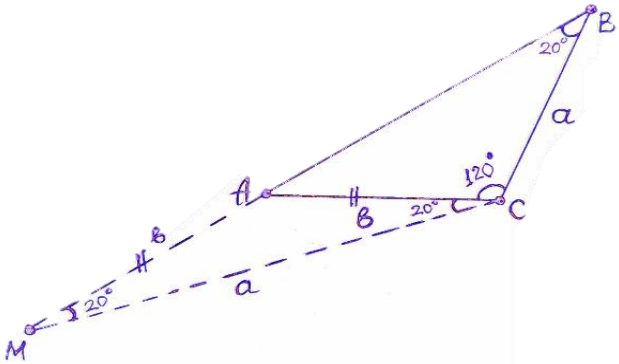


Proposed by Jafar Nikpour-Iran

Solution by Togrul Ehmedov-Azerbaijan

Using the law of cosines in △ABC, we conclude that

$$AB = \sqrt{a^2 + b^2 - 2abcos(120^0)} = \sqrt{a^2 + b^2 + ab}$$

△BCM∼△MA, $\frac{MB}{MC} = \frac{MC}{AC} \Rightarrow \frac{MA+AB}{MC} = \frac{MC}{AC}$

$$\frac{b + \sqrt{a^2 + b^2 + ab}}{a} = \frac{a}{b}, \qquad b\sqrt{a^2 + b^2 + ab} = a^2 - b^2, \qquad a^3 - b^3 = 3ab^2$$