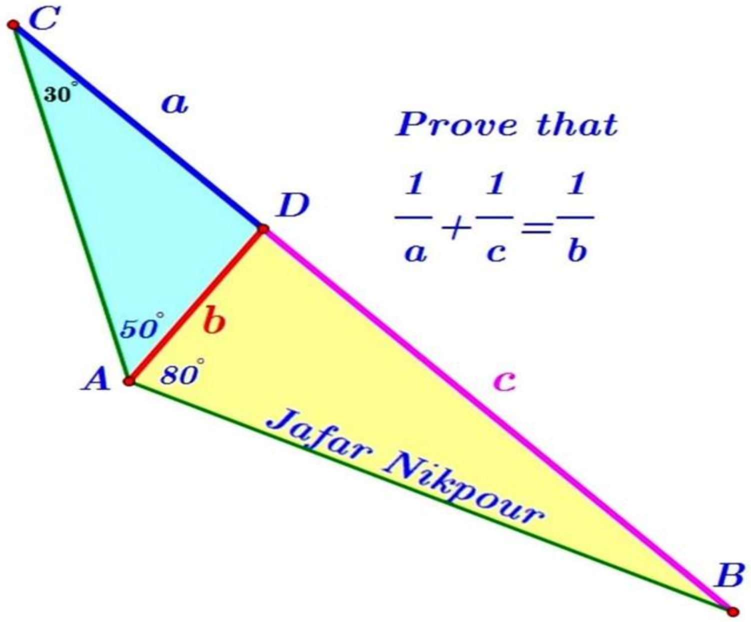
ROMANIAN MATHEMATICAL MAGAZINE



Proposed by Jafar Nikpour-Iran

Solution by Daniel Sitaru-Romania

$$m(\sphericalangle CDA) = 180^\circ - 50^\circ - 30^\circ = 100^\circ, m(\sphericalangle ADB) = 180^\circ - 100^\circ = 80^\circ$$

In ΔADC:

$$\frac{a}{sin50^\circ} = \frac{b}{sin30^\circ} \Rightarrow b = \frac{asin30^\circ}{sin50^\circ}$$

In ΔADB:

$$\frac{c}{sin80^\circ} = \frac{b}{sin20^\circ} \Rightarrow c = \frac{bsin80^\circ}{sin20^\circ} = \frac{asin30^\circ}{sin50^\circ} \cdot \frac{sin80^\circ}{sin20^\circ}$$

$$\frac{1}{a} + \frac{1}{c} = \frac{1}{b} \Leftrightarrow \frac{1}{a} + \frac{1}{\frac{asin30^\circ}{sin50^\circ} \cdot \frac{sin80^\circ}{sin20^\circ}} = \frac{1}{\frac{asin30^\circ}{sin50^\circ}} \Leftrightarrow 1 + \frac{sin20^\circ sin50^\circ}{sin30^\circ \cdot sin80^\circ} = \frac{sin50^\circ}{sin30^\circ} \Leftrightarrow$$

$$\Leftrightarrow sin30^\circ \cdot sin80^\circ + sin20^\circ sin50^\circ = sin50^\circ sin80^\circ \Leftrightarrow$$

$$\Leftrightarrow sin20^\circ sin50^\circ = sin80^\circ(sin50^\circ - sin30^\circ) \Leftrightarrow$$

$$\Leftrightarrow 2sin10^\circ cos10^\circ sin50^\circ = sin(90^\circ - 10^\circ) \cdot 2sin\frac{50^\circ - 30^\circ}{2} cos\frac{50^\circ + 30^\circ}{2} \Leftrightarrow$$

$$\Leftrightarrow 2sin10^\circ cos10^\circ sin50^\circ = 2cos10^\circ sin10^\circ cos40^\circ \Leftrightarrow$$

$$\Leftrightarrow sin50^\circ = cos40^\circ \Leftrightarrow sin50^\circ = sin50^\circ$$