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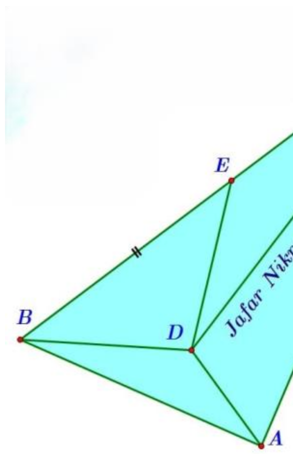
Suppose that: $\angle DBA = 20^\circ; \angle DAB = 30^\circ; \angle DBC = 40^\circ; \angle DAC = 60^\circ$

Prove that: $\angle DEC = 140^\circ$



Proposed by Jafar Nikpour – Iran

Solution by Eric - Dimitrie Cismaru – Romania

We have $\measuredangle ABC = \measuredangle DBA + \measuredangle DBC = 60^\circ, \measuredangle BAC = \measuredangle DAB + \measuredangle DBC = 90^\circ$, so ΔABC is a right triangle and $\measuredangle BCA = 30^\circ$.

On the other hand, since E is the midpoint of BC, AE is a median in a right triangle, so $[AE] = [BE] = [EC]$, and since $\measuredangle EBA = 60^\circ, \Delta BEA$ is equilateral, so we have $[BA] = [AE]$. The triangle ΔAEC is isosceles, so we have $\measuredangle EAC = \measuredangle ECA = 30^\circ$.

Therefore, $\Delta DAB \equiv \Delta DAE$, which leads us to $\measuredangle DEA = \measuredangle DBA = 20^\circ$, and since $\measuredangle AEC = 120^\circ$, we obtain $\measuredangle DEC = 140^\circ$, the conclusion.