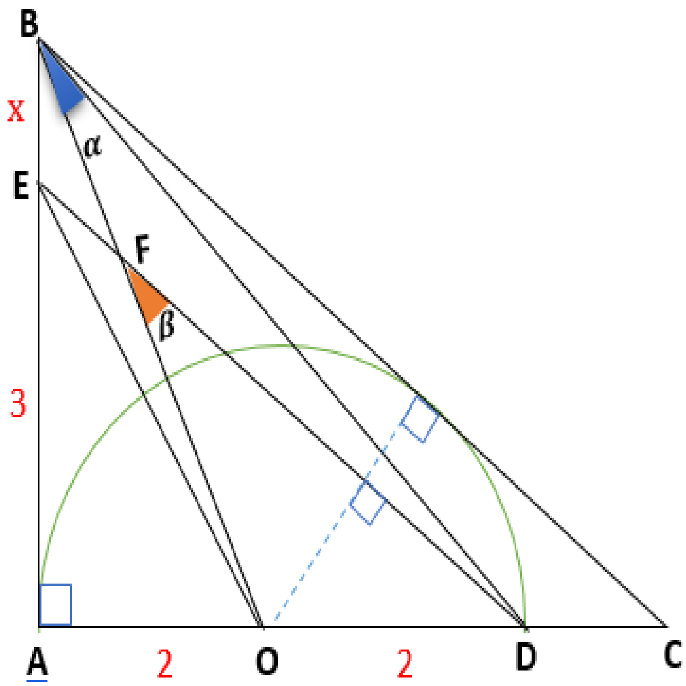
# ROMANIAN MATHEMATICAL MAGAZINE




O:= Center of the semicircle

X=?, $\alpha+\beta=?$

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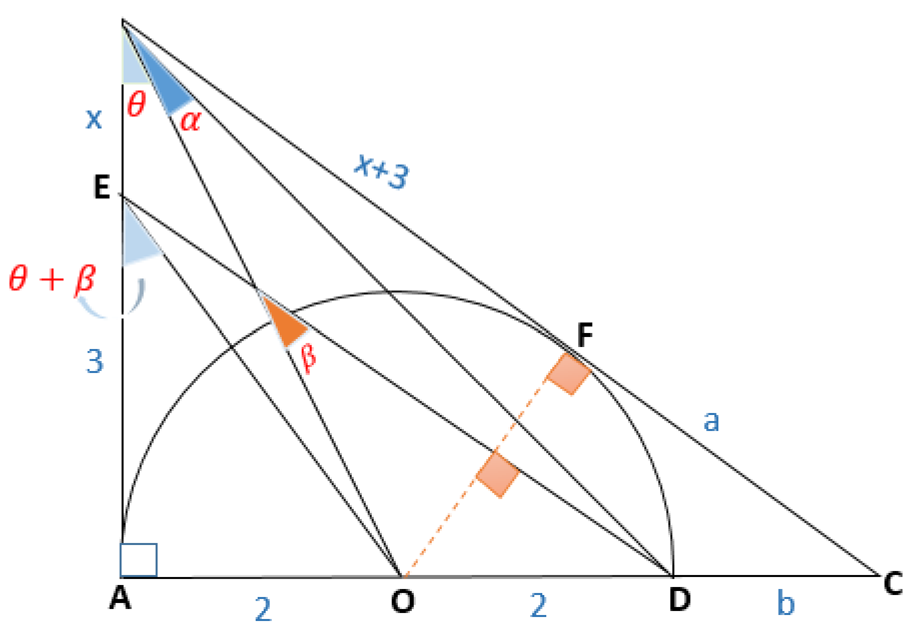
*Solution by Mirsadix Muzefferov-Azerbaijan*



$$In\ \Delta EAD\ (A=90^\circ)\ by\ teorem\ Pythagorean$$

$$ED^2=AD^2+AE^2=4^2+3^2=25\Rightarrow ED=5.$$

$$Also, ED\parallel BC\Rightarrow \Delta AED\sim\Delta ABC$$

$$\frac{5}{(3+x)+a}=\frac{3}{3+x}\Rightarrow x+3=\frac{3a}{2}\quad(1)$$

$$\dot{I}n\ ABC\quad BO\ is\ bisector\ .Then$$

$$\frac{3+x}{2}=\frac{(3+x)+a}{2+b}\Longrightarrow\frac{3a}{4}=\frac{\frac{3a}{2}+a}{2+b}\Longrightarrow b=\frac{4}{3}\quad(2)$$

$$In\ \Delta OFC\ \left(\hat{F}=90^\circ\right)\ by\ Pythagorean$$

$$a^2=(2+b)^2-2^2=\frac{100}{9}-4=\frac{64}{9}\Longrightarrow a=\frac{8}{3}\quad(3)$$

$$Therefore\ (1)\ and\ (3)\Longrightarrow x=1$$

$$In\ \Delta ABO\ \left(\hat{A}=90^\circ\right)\ \theta+\alpha=\frac{\pi}{4};$$

$$In\ \Delta AEO\ \left(\hat{A}\right)=90^\circ\Longrightarrow tan(\theta+\beta)=\frac{4}{3}$$

$$In\ \Delta AEO\ \left(\hat{A}=90^\circ\right)\Longrightarrow\tan\theta=\frac{1}{2};$$

$$\frac{4}{3}=\tan(\theta+\beta)=\frac{\tan\theta+\tan\beta}{1-\tan\theta\tan\beta}=\frac{\frac{1}{2}+\tan\beta}{1-\frac{1}{2}\tan\beta}\Longrightarrow\tan\beta=\frac{1}{2}$$

$$1=\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}=\frac{\frac{1}{2}+\tan\alpha}{1-\frac{1}{2}\tan\alpha}\Longrightarrow\tan\alpha=\frac{1}{3}$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}=1\Longrightarrow\alpha+\beta=\frac{\pi}{4}$$