ROMANIAN MATHEMATICAL MAGAZINE

İn $\Delta ABC, m(BAC) = 90°$, $AD \perp BC, D \in (BC), R, R_1, R_2$ – circumradii and r, r_1, r_2 – inradii of $\Delta ABC, \Delta ABD, \Delta ACD$. Prove that

$$\frac{R - R_1}{R + R_1} + \frac{R - R_2}{R + R_2} = \frac{r - r_1}{r + r_1} + \frac{r - r_2}{r + r_2}$$

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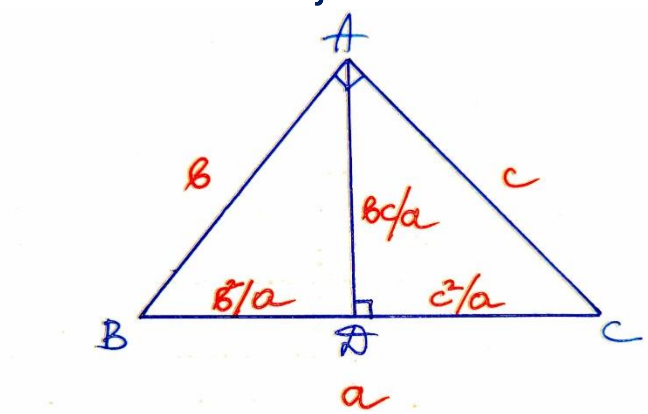
Solution by Togrul Ehmedov-Azerbaijan



$$|AB| = b, |AC| = c, |BC| = a, |BD| = \frac{b^2}{a}, |CD| = \frac{c^2}{a}, |AD| = \frac{bc}{a}$$

$$\text{We know that } R = \frac{a}{2}, R_1 = \frac{b}{2}, R_2 = \frac{c}{2} \text{ and}$$

$$r = \frac{b + c - a}{2}, r_1 = \frac{b}{a}r, r_2 = \frac{c}{a}r$$

$$\frac{R - R_1}{R + R_1} + \frac{R - R_2}{R + R_2} = \frac{r - r_1}{r + r_1} + \frac{r - r_2}{r + r_2} \Rightarrow \frac{\frac{a}{2} - \frac{b}{2}}{\frac{a}{2} + \frac{b}{2}} + \frac{\frac{a}{2} - \frac{c}{2}}{\frac{a}{2} + \frac{c}{2}} = \frac{r - \frac{b}{a}r}{r + \frac{b}{a}r} + \frac{r - \frac{c}{a}r}{r + \frac{c}{a}r} \Rightarrow$$

$$\Rightarrow \frac{a - b}{a + b} + \frac{a - c}{a + c} = \frac{a - b}{a + b} + \frac{a - c}{a + c}$$