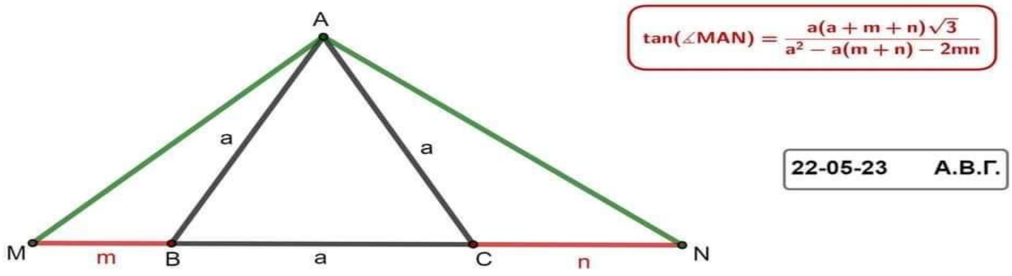
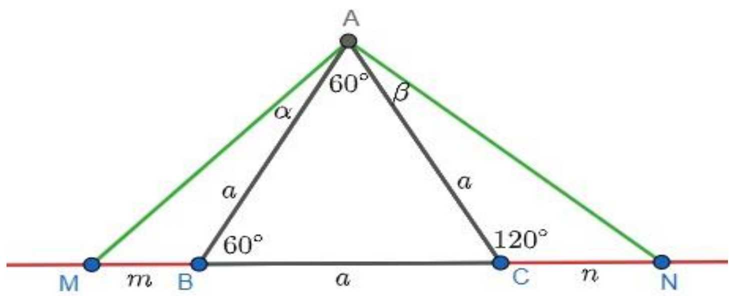
ROMANIAN MATHEMATICAL MAGAZINE



With reference to diagram, prove that :

$$\tan(\measuredangle MAN) = \frac{\sqrt{3}.4a(a+m+n)}{4(a^2 - a(m+n) - 2mn)}$$

Proposed by Thanasis Gakopoulos-Farsala-Greece
Solution by Soumava Chakraborty-Kolkata-India



Via cotangent rule : $(m+a)\cot 60° = m\cot\alpha - a\cot 60°$

$$\Rightarrow \frac{a+m}{\sqrt{3}} = \frac{m}{\tan\alpha} - \frac{a}{\sqrt{3}} \Rightarrow \tan\alpha \overset{(*)}{=} \frac{\sqrt{3}m}{2a+m}$$

Also, via cotangent rule : $(a+n)\cot 120° = a\cot 60° - n\cot\beta$

$$\Rightarrow \frac{a+n}{-\sqrt{3}} = \frac{a}{\sqrt{3}} - \frac{n}{\tan\beta} \Rightarrow \tan\beta \overset{(**)}{=} \frac{\sqrt{3}n}{2a+n}$$

$$\therefore \tan(\measuredangle MAN) = \frac{\tan\alpha + \tan\beta + \tan 60° - \tan\alpha.\tan\beta.\tan 60°}{1 - \tan\alpha.\tan\beta - \tan\alpha.\tan 60° - \tan\beta.\tan 60°} \overset{\text{via } (*),(**)}{=}$$

$$\frac{\frac{\sqrt{3}m}{2a+m} + \frac{\sqrt{3}n}{2a+n} + \sqrt{3} - \frac{\sqrt{3}m}{2a+m}.\frac{\sqrt{3}n}{2a+n}.\sqrt{3}}{1 - \frac{\sqrt{3}m}{2a+m}.\frac{\sqrt{3}n}{2a+n} - \frac{\sqrt{3}m}{2a+m}.\sqrt{3} - \frac{\sqrt{3}n}{2a+n}.\sqrt{3}} = \frac{\sqrt{3}.4a(a+m+n)}{4(a^2 - am - an - 2mn)}$$

$$\Rightarrow \tan(\measuredangle MAN) = \frac{\sqrt{3}.4a(a+m+n)}{4(a^2 - a(m+n) - 2mn)} \text{ (QED)}$$