ROMANIAN MATHEMATICAL MAGAZINE

**In $\Delta ABC$ the following relationship holds:**

$$2\sum_{cyc}\frac{m_a}{h_a} \geq \frac{2s}{\sqrt[3]{abc}} + \frac{1}{\sqrt[3]{h_a h_b h_c}}\sum_{cyc} h_a$$

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**By Tereshin and AM − GM inequalities, we have**

$$2\sum_{cyc}\frac{m_a}{h_a} \geq 2\sum_{cyc}\frac{b^2+c^2}{4Rh_a} = \sum_{cyc}\frac{b^2+c^2}{bc} = \sum_{cyc}\frac{a}{b} + \sum_{cyc}\frac{h_a}{h_b} =$$

$$= \sum_{cyc}\frac{1}{3}\left(\frac{a}{b}+\frac{a}{b}+\frac{b}{c}\right) + \sum_{cyc}\frac{1}{3}\left(\frac{h_a}{h_b}+\frac{h_a}{h_b}+\frac{h_b}{h_c}\right) \geq \sum_{cyc}\sqrt[3]{\frac{a^2}{bc}} + \sum_{cyc}\sqrt[3]{\frac{{h_a}^2}{h_b h_c}} =$$

$$= \frac{2s}{\sqrt[3]{abc}} + \frac{1}{\sqrt[3]{h_a h_b h_c}}\sum_{cyc} h_a.$$

**Equality holds iff $\Delta ABC$ is equilateral.**