ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC, \Delta A'B'C'$ the following relationship holds:

$$(a+a')(b+b')(c+c') \geq 32\sqrt{RR'FF'} + 4\left(\sqrt{RF} - \sqrt{R'F'}\right)^2$$

Proposed by Daniel Sitaru-Romania

Solution by Tapas Das-India

$$(a+a')(b+b')(c+c') =$$

$$= abc + a'b'c' + (a'bc + ab'c + abc' + a'b'c + ab'c' + a'bc') \geq$$

$$\overset{AM-GM}{\geq} abc + a'b'c' + 6\sqrt[6]{a'bc \cdot ab'c \cdot abc' \cdot a'b'c \cdot ab'c' \cdot a'bc'} =$$

$$= abc + a'b'c' + 6\sqrt{abc \cdot a'b'c'} = 4RF + 4R'F' + 6\sqrt{4RF \cdot 4R'F'} =$$

$$= 4\left(\sqrt{RF} - \sqrt{R'F'}\right)^2 + 8\sqrt{RR'FF'} + 24\sqrt{RR'FF'} = 32\sqrt{RR'FF'} + 4\left(\sqrt{RF} - \sqrt{R'F'}\right)^2$$

Equality holds for $a = b = c = a' = b' = c'$.